期末综合小练（15）

1. $cos170°sin80° + cos20°sin10° =$ (　)

A. $-\frac{\sqrt{3}}{2}$　　B. $\frac{\sqrt{3}}{2}$　　C. $-\frac{1}{2}$　　D. $\frac{1}{2}$

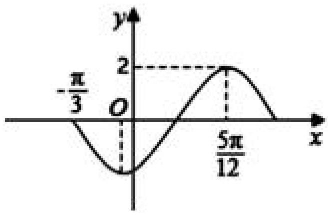
2. 若函数$f(x) = \begin{cases} -x^{\frac{1}{3}}, x \leq -1 \\ x + \frac{2}{x} - 7, x > -1 \end{cases}$，则$f[f(-8)] =$ (　)

A. -2　　B. 2　　C. -4　　D. 4

3. 若向量$\vec{a}$，$\vec{b}$满足$|\vec{a}| = \sqrt{3}$，$|\vec{b}| = 2$，$\vec{a} \perp (\vec{a} - \vec{b})$，则$\vec{a}$与$\vec{b}$的夹角为(　)

A. $\frac{\pi}{2}$　　B. $\frac{2\pi}{3}$　　C. $\frac{\pi}{6}$　　D. $\frac{5\pi}{6}$

已知函数$f(x) = Asin(\omega x + \varphi)(A > 0, \omega > 0, |\varphi| < \frac{\pi}{2})$的部分图象如图所示.



(1)求$f(x)$的解析式；

(2)将$y = f(x)$图象上所有点向左平行移动$\frac{\pi}{12}$个单位长度，得到$y = g(x)$图象，求函数$y = g(x)$在$[0, \pi]$上的单调递增区间.

15 答案和解析

1. 【答案】D

【解析】解：$cos70°sin80° + cos20°sin10° = sin20°cos10° + cos20°sin10°$

$= \sin(20° + 10°) = sin30° = \frac{1}{2}$.

2. 【答案】C 解：∵函数$f(x) = \begin{cases} -x^{\frac{1}{3}}, x \leq -1 \\ x + \frac{2}{x} - 7, x > -1 \end{cases}$，

又∵$-8 < -1$，∴ $f(-8) = -(-8)^{\frac{1}{3}} = 2$，∵ $2 > -1$，

∴ $f[f(-8)] = f(2) = 2 + \frac{2}{2} - 7 = -4$.

3. 【答案】C

解：设$\vec{a}$与$\vec{b}$的夹角为θ，$\theta \in [0, \pi]$，∵ $|\vec{a}| = \sqrt{3}$，$|\vec{b}| = 2$，$\vec{a} \perp (\vec{a} - \vec{b})$，

∴ $\vec{a}^2 - \vec{a} \cdot \vec{b} = 3 - 2\sqrt{3}\cos\theta = 0$，∴ $\cos\theta = \frac{\sqrt{3}}{2}$，∴ $\theta = \frac{\pi}{6}$. 故选 C.

4. 【答案】解：(1)由图象可知，$A = 2$，周期$T = \frac{4}{3}[\frac{5\pi}{12} - (-\frac{\pi}{3})] = \pi$，

∴ $\frac{2\pi}{|\omega|} = \pi$，$\omega > 0$，则$\omega = 2$，从而$f(x) = 2sin(2x + \varphi)$，代入点$(\frac{5\pi}{12}, 2)$，

得 $\sin(\frac{5\pi}{6} + \varphi) = 1$，则$\frac{5\pi}{6} + \varphi = \frac{\pi}{2} + 2k\pi$，$k \in Z$，即$\varphi = -\frac{\pi}{3} + 2k\pi$，$k \in Z$，

又$|\varphi| < \frac{\pi}{2}$，则$\varphi = -\frac{\pi}{3}$，∴ $f(x) = 2sin(2x - \frac{\pi}{3})$，

(2)由(1)知$f(x) = 2sin(2x - \frac{\pi}{3})$，

因此$g(x) = 2sin[2(x + \frac{\pi}{12}) - \frac{\pi}{3}] = 2sin(2x - \frac{\pi}{6})$，

令 $2k\pi - \frac{\pi}{2} \leq 2x - \frac{\pi}{6} \leq 2k\pi + \frac{\pi}{2}$，$k \in Z$，可得：$k\pi - \frac{\pi}{6} \leq x \leq k\pi + \frac{\pi}{3}$，$k \in Z$，

由$[k\pi - \frac{\pi}{6}, k\pi + \frac{\pi}{3}] \cap [0, \pi] = [0, \frac{\pi}{3}] \cup [\frac{5\pi}{6}, \pi]$，单调递增区间为$[0, \frac{\pi}{3}]$，$[\frac{5\pi}{6}, \pi]$.